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Cover

Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, Gift of Philip Hofer, MS 1984.463. fol. 61[r]: This folio shows in the middle at the right the riddle text in large letters in *thulūth* calligraphy. Between the five lines of this riddle is a part of an Arabic philosophical work in *naskhī* comprising three lines in each piece. In red, numbers and words are placed mostly below individual words of the riddle referring to letter magic. Around this centre piece, two brief Persian texts in *nastaʿlīq*, an Arabic table, and a triangular diagram between lines of an Arabic explanation can be found. Both Arabic pieces are written in *naskhī*. The Persian text above the table introduces the lunar mansions, which the table enumerates. The Persian text in the left margin, entitled „A gem on theoretical philosophy about true speech", deals with themes from *kalām*. The triangular diagram with its surrounding Arabic text treats the cosmological division of the universe in Muslim terms, beginning with God's throne and descending through the Ptolemaic planetary sphere to the four Aristotelian spheres of the sublunar world to the underworld.

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INTRODUCTION

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Article

The Mathematical Sciences in Ẓāhir al-Dīn Muḥammad Ibrāhīm's Collection of Riddles from 1070 H/1660 CE

Sonja Brentjes | Berlin

In this paper, I shall discuss the mathematical sciences as presented in the collection of Persian and Arabic texts, tables and diagrams that form Ẓāhir al-Dīn Ibrāhīm Muḥammad's work *Nihāyat al-aqdām fī ṭawr al-kalām* ('The Farthest Points Reached in the Science of Theology').[1] I drew my findings from four of the extant multiple-text manuscripts that contain this collection of items: MS Cambridge, Mass., Harvard Art Museums, Sackler Art Museum, 1984.463 (referred to from now on as the 'Sackler manuscript' in this paper); MS Istanbul, Süleymaniye Library, Ayasofya 4785 (the 'Ayasofya manuscript'); MS London, British Library, Or. 12974 (the 'British Library manuscript'); and MS Tehran, Malik Library, 868 (the 'Malik 868 manuscript'). In chronological terms, the last one mentioned is the oldest of the four copies (1085 H/1674–75 CE), followed by the Ayasofya (1086 H/1675–76 CE), British Library (1089 H/1678–79 CE) and Sackler (1098 H/1686–87 CE) manuscripts.

The first paper in this volume discusses Ẓāhir al-Dīn's work and analyses the relationship between these four manuscripts and a fifth one, MS Tehran, Malik Library, 1517.[2] Its authors argue that each manuscript may have been copied for an individual client with specific interests. The eighth paper in this volume investigates an important aspect of the use of mathematical knowledge to solve the riddles constituting the focus of Ẓāhir al-Dīn's collection.[3] My own paper is more modest in its goals, however. It first discusses the classification of the mathematical sciences and its contradictions as presented by Ẓāhir al-Dīn. The second part surveys the presentation of knowledge from two of the four fundamental mathematical sciences – number theory and geometry – and four of the subdisciplines in texts, tables and diagrams (calculation systems, algebra, surveying and geography). The third part addresses the difficult question of how the scribe who wrote all five manuscripts went about adding the texts, tables and diagrams and how he used the space available on each page in this process. The material that Ẓāhir al-Dīn considered as belonging to the mathematical sciences, such as optics, planetary models, sand divination and lettrism, is not discussed here, partly because the latter is at the heart of Mousavi and Bohloul's contribution to this volume and partly because its specific functions in Ẓāhir al-Dīn's collection need further study.

I have followed Mousavi and Bohloul's choice of translating the term *majlis* as 'banquet'[4] in my paper because I assume that riddle-solving was not just intended as an intellectual exercise, but as an opportunity to socialise with appreciated companions in an enjoyable atmosphere with food, drink and music and poetic recitations performed by highly skilled artists.

First of all, my analysis of the aforementioned parts of the manuscripts has shown that they primarily served as a 'representation' of disciplinary knowledge and 'decorate' the riddle text. They fill a number of margins and interlinear spaces around or in the latter without being needed to solve any of its riddles. Secondly, the parts of the investigated material that are present in all four manuscripts seem to have been planned for inclusion in the process of compiling the (lost) original collection commissioned by Ẓāhir al-Dīn. Thirdly, some components were apparently only added later, in the process of copying the collection for other interested clients.

The highly challenging character of the collection makes it very difficult to draw any reliable conclusions. My interpretations of specific features of the analysed mathematical texts, tables and diagrams should therefore

[1] For translations of this rhymed title see also Pourjavady and Rahimi-Riseh, Mousavi and Bohloul, and Kia in this volume.

[2] Pourjavady and Rahimi-Riseh 2024. I thank Reza Pourjavady for providing me with a copy of the fifth manuscript to help me understand the production process behind the two manuscripts preserved at Malik Library in Tehran.

[3] Mousavi and Bohloul 2024.

[4] See the contribution of Mousavi and Bohloul in this volume.

be understood as preliminary proposals that are in need of further investigation.

1. The classification of the mathematical sciences in Ẓāhir al-Dīn's collections

Ẓāhir al-Dīn's classification of the mathematical sciences follows the centuries-old division of them into the four fundamental disciplines of neo-Platonic school teaching and a set of derived branches that started to emerge in Antiquity and later multiplied and diversified in scholarly debates within Islamicate societies. Ẓāhir al-Dīn presents two classifications – one in the narrative introduction to his *Talkhīṣ Asās al-iqtibās*,[5] a summary of Naṣīr al-Dīn Ṭūsī's (d. 672 H/1274 CE) work on logic, *Asās al-iqtibās*, and the other in a tabular arrangement immediately after that.

The order of the four fundamental mathematical sciences differs in the two classifications, however. Readers at the time would have expected such an arrangement to be coherent, in particular whenever the definitions follow one another. One possible interpretation of this mixed state of affairs could be that the order of the four sciences did not matter very much to Ẓāhir al-Dīn. This would be surprising, though, because the first sequence is in tune with his main intellectual message in this collection – the primeval dominance of the One (*wāḥid*)[6] – whereas the second sequence contradicts this message.

Ẓāhir al-Dīn's first, very brief classification agrees in principle with the standard classification of Late Antique neo-Platonic school teaching, which started the sequence with number theory, followed by geometry, astronomy and music. There are philosophical reasons for this order, which moves from the highest to the lowest in status. The highest is number theory because the One is not a number, but the root and origin of all natural numbers; it is the abstract One. As such, it only has one property and thus ranges above the point, which has two properties, namely being indivisible, and thus one, and having a position. Consequently, number theory (*arithmētikē*) comes before geometry. The two disciplines are more noble than the two that follow because astronomy and music deal with material objects, while arithmetic and geometry are concerned with abstract phenomena, that is, matters of the mind. The ranking between astronomy and music follows from the hierarchy between the celestial and the terrestrial realms.

Ẓāhir al-Dīn's third science in this first classification is not astronomy as a whole, however, but *hayʾa*, the science of the configuration (of the universe), which modern translators usually identify with planetary theory or mathematical cosmography.[7] *Hayʾa* fits into the sequence of fundamental mathematical sciences well, but it does not, in the general understanding of the word, serve as an overarching term for the other parts of the astral sciences, such as the science of astronomical handbooks (*ʿilm al-azyāj*) or the science of timekeeping (*ʿilm al-mīqāt*).

With respect to its terminological peculiarity, Ẓāhir al-Dīn's choice of wording agrees with that of Ibn Sīnā (d. 428 H/1037 CE) in his early work *Risāla fī aqsām al-ʿulūm al-ʿaqliyya*.[8] In Ibn Sīnā's scheme of things, the term defines astronomy in the ancient sense since he describes it as the knowledge taught in Ptolemy's *Almagest*.[9] This is also the case for Ẓāhir al-Dīn, who tells his readers before providing the formal classification that one studies 'numbers and music with Pythagoras, geometry with Euclid and *hayʾa* with Ptolemy'.[10]

Ẓāhir al-Dīn's second scheme reorders Ibn Sīnā's sequence, rendering it philosophically meaningless. It starts with geometry (*handasa*), followed by number theory using the transliterated Greek term *arithmāṭīqī*, astronomy (*usṭrulūmiyā* [*sic*]) and music (*mūsīqā*).[11] The change of position between geometry and number theory is the feature of this classification that contradicts the vizier's lettrist message, because the point, being the one in geometry, has the two properties mentioned above. This description is the definition of the point as found in Naṣīr al-Dīn Ṭūsī's version of Euclid's *Elements*. Since Ṭūsī's version had become the most widespread edition of the *Elements* and had either been taught in the madrasa or in other teaching environments since the early eighth century H/fourteenth century CE at least, in all likelihood Ẓāhir al-Dīn was familiar with this definition of the point and its difference to the definition of the numerical one, which only had one property.

This idea of the two different 'ones' was not only taught in the *Elements*, but in texts committed to the neo-Pythagorean

[5] Another form of the title is *Qabasāt-i Asāsiyya*. See the contribution of Pourjavady and Rahimi-Riseh in this volume.

[6] See the contribution of Mousavi and Bohloul in this volume.

[7] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6a, first half, lines 10–11.

[8] Ibn Sīnā, *Rasāʾil*, 1298 H/1881, 76.

[9] Ibn Sīnā, *Rasāʾil*, 1298 H/1881, 76.

[10] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 5a, second half, lines 3–5.

[11] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 7b.

heritage, such as the *Rasāʾil* ('treatises') of the *Ikhwān al-Ṣafāʾ* (the 'Encyclopaedia of the Brethren of Purity', fourth century H/tenth century CE) and Shams al-Dīn Āmulī's (d. 753 H/1352 CE) Persian encyclopaedia *Nafāyis al-funūn fī ʿarāyis al-ʿuyūn*.[12] Since neo-Pythagorean teachings about the One and natural numbers had a primary place in Ẓāhir al-Dīn's world view as presented in his collection, his incongruence is vexing.

It is all the more puzzling as the vizier claims to have followed this definition in his second presentation of the scientific system of the aforementioned Persian encyclopaedia by Shams al-Dīn Āmulī.[13] This is not the case, however. Āmulī grouped geometry and astronomy together, followed by the pairing of number theory and music, thereby restoring some kind of coherence within the sequence because of the disciplinary interrelationship between the two groups.[14] Not even Ẓāhir al-Dīn's usage of transliterated Greek terms is something that he may have appropriated directly from Āmulī's encyclopaedia because the two authors differ in their respective designations for geometry and astronomy.[15] Thus, Āmulī's classification clearly does not tally with Ẓāhir al-Dīn's second scheme, which orders the four fundamental sciences the same way as Naṣīr al-Dīn Ṭūsī does in his *Akhlāq-i naṣīrī*.[16]

Ṭūsī's classification does not agree fully with the text provided by Ẓāhir al-Dīn either, though. First of all, Ṭūsī's description of the content of the four fundamental mathematical sciences differs from that found in Ẓāhir al-Dīn's text. Second, Ṭūsī's classification does not include any of the occult branches of science that Ẓāhir al-Dīn mentions. Ṭūsī even explicitly excludes the only part of astrology mentioned, namely judicial astrology, from 'the science of the stars' as a mathematical science; the text sorts it as one of the branches of natural philosophy instead.[17]

All in all, it is clear that at least four different orders of the fundamental mathematical sciences were propagated by different authors in the Persianate world. The only discipline that never seems to have been put first was music. This hierarchy suggests that music was of little importance in the ongoing discussions on the names, positions and hence status of the four fundamental mathematical sciences. Such a view is supported by the very small number of remarks and one part of a treatise on music that Ẓāhir al-Dīn chose to include in his collection.

The set of branch disciplines in Ẓāhir al-Dīn's second classification contains a surprise of a different kind. It includes more specific content than usually found in short classifications. He describes geometry as the rules about magnitudes and figures and the knowledge of spheres, the rest of (what belongs to) the 'Middle (Books)', perfect and deficient cones (parabolas and ellipses), boundaries and the foundational principles (axioms). Then he informs his readers that some books (without specifying which) contain postulates. Although other ancient Greek mathematical works also contain postulates, it seems likely that Ẓāhir al-Dīn was referring to Euclid's *Elements* here.

Since Ẓāhir al-Dīn adds that he has mentioned the postulates already, which is not the case, it is obvious that he copied his summary from some other source. Ṭūsī's survey cannot have served him here due to its brevity, however. A comparison between Ẓāhir al-Dīn's text and Āmulī's third chapter of the second part, which discusses geometry, shows that it did not serve as one of Ẓāhir al-Dīn's sources either. Āmulī did not merely present this discipline through his Persian rendering of Book I of Euclid's *Elements*, but rather opted for a fundamentally different entry: while Ẓāhir al-Dīn's source discusses the discipline from the perspective of its content, Āmulī introduces it according to its ancient Greek authors and their texts.[18]

It thus remains unclear which classificatory booklet or encyclopaedia Ẓāhir al-Dīn employed as a source in his own description of geometry. That said, it should be noted that the information he provides is imprecise and disorderly. It moves unsystematically between different aspects of the *Elements*, referring in between to the 'Middle Books', which encompass astronomical and geometrical works by ancient Greek and medieval Muslim scholars and Apollonius of Perga's *Conics*. After the reference to the postulates, Ẓāhir al-Dīn lists

12 Āmulī, *Nafāyis al-funūn*, ed. Shaʿrānī and Miyanjī 1379 HS/2002, vol. 3, 3.

13 MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6b.

14 Vesel 1986, 39–40.

15 Āmulī, *Nafāyis al-funūn*, ed. Shaʿrānī and Miyanjī 1379 HS/2002, vol. 3, 2, 26. Āmulī names geometry in the heading *ʿilm-i ustuqussāt*, while Ẓāhir al-Dīn's heading uses the standard term *handasa*. Astronomy simply appears in Āmulī's heading in its Arabic transliteration, while Ẓāhir al-Dīn opted for the more elaborate title '... tawṣīḥāt abwāb astru<n>ūmiyā'. In addition, both copies of Ẓāhir al-Dīn's *Nihāyat al-aqdām* that contain this classification say *usṭrulūmiyā* instead of *usṭrunūmiyā*.

16 Stephenson 1923, 329–38; Ṭūsī, *Akhlāq-i nāṣirī*, ed. Mīnuvī and Ḥaydarī 1356 HS/1977, 39.

17 Stephenson 1923, 332–33; Ṭūsī, *Akhlāq-i nāṣirī*, ed. Mīnuvī and Ḥaydarī 1356 HS/1977, 39–40.

18 Āmulī, *Nafāyis al-funūn*, ed. Shaʿrānī and Miyanjī 1379 HS/2002, vol. 3, 2.

three themes that do not belong to the standard sequence of definitions in Book I of Euclid's *Elements*. The first is the five (Platonic) solids. These are treated in Book XIII of the *Elements*. Then the existence of different cases is mentioned. This is probably a reference to the proofs for specific cases rather than general ones, such as a scalene or a rectangular triangle rather than a triangle as such, irrespective of the lengths of its sides or the size of its angles. There are two procedures used in Arabic and Persian versions of the *Elements*, but they are not explicitly discussed as two different methodological approaches in most of them. Thirdly, a reference to spherical geometry follows, a subject which is not part of the *Elements*. Afterwards, the author returns to the one-dimensional objects (straight and curved lines) of Book I of the *Elements*, as discussed in commentaries.[19]

This state of affairs indicates that Ẓāhir al-Dīn was neither well trained in geometry nor a particularly methodical thinker. This is rather surprising as his reputation as an expert in the mathematical sciences was pointed out by several travellers from Western Europe.[20]

The next box in Ẓāhir al-Dīn's second classification deals with arithmetic. It differs significantly from the one on geometry. It is systematic both in its individual topics and the hierarchy of the disciplinary organisation of this field of knowledge. The character of the third box, entitled 'Parts of astronomy', lies between that of geometry and arithmetic. It is more ordered than the box on geometry, but shares some confusion with it in terms of its thematic presentation. It begins with a reference to the configuration of the orbs (i.e., planetary theory), followed by references to the division of the zodiac and the mansions, which most likely signify the asterisms through which the moon moves every month. But then come elements needed in the manufacture of astrolabes, such as the projection of circles and the different placements of curves. Without relating these two topics to instruments, the text moves on to the standard theme of sizes, distances and movements of planets followed by pointers to elements of a planetary orbit such as perigee and apogee and the eclipse dragon (i.e., the lunar nodes). A few lines later, Ẓāhir al-Dīn mentions the astronomical handbooks as a field of astronomy along with eclipses and their causes and occurrences. Thereafter, he makes reference to proof of the sphericity of the Earth and the entire universe. The remaining part of the list includes the task of measuring the height of mountains, the reasons for the differences between the duration of days and nights, the oblique rising of celestial bodies, all according to 'the understanding of the *Almagest*'. The last box is dedicated to music. It combines a few references to the theory of proportions with a longer enumeration of musical subjects, including instruments and entertaining sessions. It ends, somewhat surprisingly, at least for me, with a reference to the melodies produced by the movements of the celestial spheres.[21] This is most likely a further element appropriated from texts incorporating ideas from the Late Antique neo-Pythagorean tradition. Further research is needed here.

The first three foundational mathematical sciences include short references to their branch sciences at the end, which Ẓāhir al-Dīn calls 'accessories' (*lāḥiqa*). In the box on geometry, he enumerates optics and burning mirrors, surveying, the construction of observational instruments, instruments for playing and defence, military instruments, and tools for moving heavy objects and determining the time, proportional compasses, and an 'instrument of reflection for transforming a small plane figure into a big one'. Exactly what kind of instrument is meant in this last case is another enigma regarding this collection. Ẓāhir al-Dīn also lists quantities related to weights for metals, gemstones and waters and discusses what they might reveal about their qualities and their relationships to each other.[22] This is a surprisingly rich enumeration of themes that were not documented in the extant geometrical manuscripts of the time.

The text in the box on number theory identifies this discipline first as (knowledge of) the essence of numbers, the quantities that arise according to the natural order from the one, which comes before the two, and knowledge of the ratios and relations between two of them. Ẓāhir al-Dīn adds the three-dimensional figurative numbers, the quality of operations in calculation, and the determination of square and cubic roots and of unknowns by pointing to a variety of methods including analysis and algebra. Interestingly, he declares the construction of magic squares (*al-alwāḥ al-wafqiyya*) as the noblest goal of this discipline (albeit without any further explanation). Under the subdisciplines, he enumerates letter magic, the determination of names, tools for solving numerical and letter equalities, and matters

[19] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6b.

[20] See the contribution of Mousavi and Bohloul in this volume.

[21] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6b.

[22] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6b.

attributed to the Pythagoreans, among other subjects. He concludes with the philosophical idea found in Nikomachos of Gerasa's *Introduction to Arithmetic* that the form of numbers in the soul is congruent to the form of existent beings in matter – an idea which the seeker of knowledge considers the root of the sciences and the foundation of philosophy (or wisdom [*ḥikma*]).[23]

The subdisciplines of astronomy comprise the rules concerning the revolutions, conjunctions and aspects, the fixation of the image of the Earth and its zones called climes, their limits, the seas and winds, the routes and kingdoms, and the determination of the longitudes and latitudes of countries (*aqālīm*). The inclusion of narrative geography – known under the rubric of 'the routes and kingdoms' (*al-masālik wa-l-mamālik*) – in this list is of a much older date and can already be found in the first half of the fourteenth century in Āmulī's encyclopaedia, if not before.[24]

In addition, Ẓāhir al-Dīn subordinates several disciplines to astronomy as branches that a modern reader would not identify as mathematical or astronomical, including physiognomy, geomancy and oneiromancy.[25] While geomancy was classified as a mathematical science as early as the twelfth century, it is unclear when and in which context physiognomy and oneiromancy were subsumed under astronomy.[26] Music is the only representative of the four fundamental mathematical disciplines that is not expressly endowed with any branches, which is why its parts were discussed under the general heading of the main discipline.

Another little-known feature in classifications of the sciences by authors of the post-classical period is the overlap between parts of natural philosophy and astronomy as found in Ẓāhir al-Dīn's source. Although the Earth and its divisions and measurements are squarely placed within astronomy, the proof of the sphericity of the Earth and the universe – an important aspect of Ptolemy's *Almagest* – does not appear in the box dedicated to this mathematical science, but rather in natural philosophy under the Aristotelian-type heading 'the heavens and the world (or universe)' (*fī l-samāʾ wa-l-ʿālam*).[27]

[23] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6b.

[24] Vesel 1986, 40.

[25] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 6b.

[26] Cf. Vesel 1986, 36–37 and 39–40 and Melvin-Koushki 2021, 403.

[27] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 7a.

This second classification of the sciences in Ẓāhir al-Dīn's summary of Ṭūsī's work on logic is an interesting text in itself, an as yet unknown text on this important field of scholarly inquiry. Its most important features are not its placing of the different kinds of occult disciplines among the mathematical sciences or under natural philosophy, but the wealth of detail presented in the more classical parts of these two fields of theoretical philosophy. This might suggest that the boundaries between the mathematical sciences and natural philosophy were much more permeable than expected, at least for Ẓāhir al-Dīn, especially in light of the contemporary trend of including a number of occult sciences in the mathematical sciences. Moreover, the contents of the boxes dedicated to the four fundamental mathematical sciences highlight the fact that these changes in the order of the disciplines and their classification did not apply to all four of them in the same way, but only to two of them: number theory and astronomy. This might invite us to rethink the issue of 'mathematicalisation' as a feature linked more to celestial matters and natural numbers than to a shift in mathematical epistemology as a whole.[28]

2. On some of the mathematical content in Ẓāhir al-Dīn's collection

The number of mathematical texts, tables and diagrams varies between the four manuscripts, but the differences are rather insignificant. Depending on what is included in a count, the total figure can vary, but it is around 50 for all of them, excluding the texts and tables on physiognomy and geomancy, but including the geographical tables, diagrams and brief textual snippets the manuscripts contain. The majority of these items concern topics associated with the astral sciences, including calendars. I have excluded this astral material from my survey here because it is not one of my fields of expertise. I shall focus on number theory, algebra, arithmetical operations, geometry and geography in this paper.

2.1 Number theory

Since the relationship between numbers and letters and thus arithmetic and lettrism has already been discussed by Mousavi and Bohloul in their contribution to this volume, I have chosen to focus on Ẓāhir al-Dīn's contributions to even and odd numbers, amicable numbers and figurative numbers. All three topics were a standard content of Arabic, Persian and Turkish school texts on arithmetic, in particular in surveys on systems of calculation, algebra and number theory.

[28] Melvin-Koushki 2017.

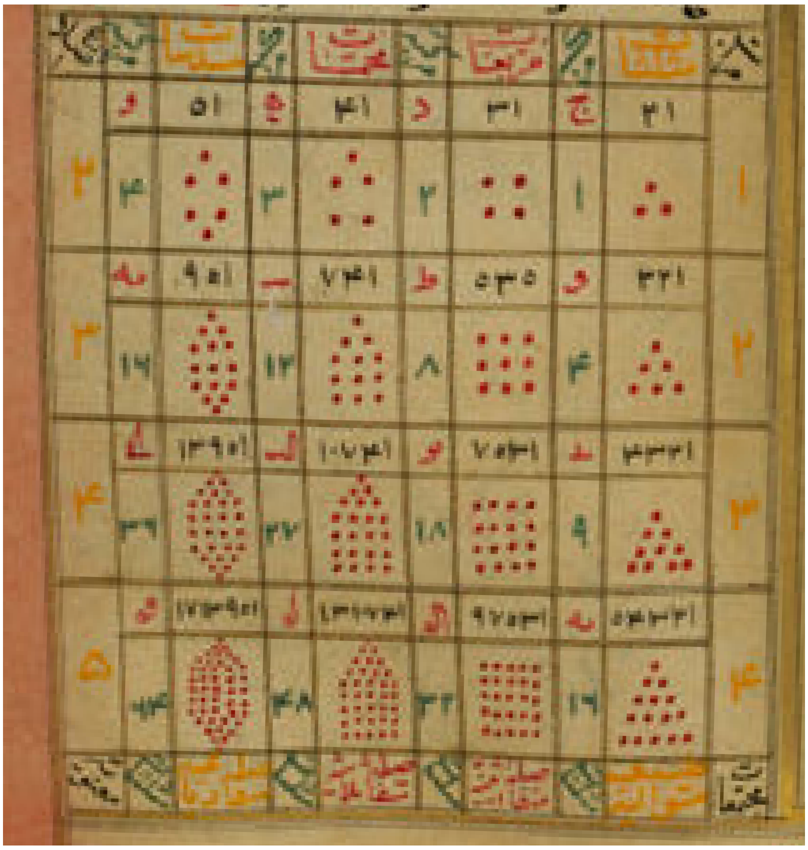

Fig. 1a: MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 51a., detail: table of figurative numbers.

Fig. 1b: MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 51a.

Table 1: Figurative numbers, MS Cambridge, Mass., Harvard Arts Museums/Arthur M. Sackler Museum, 1984.463, fol. 51a.

rank	triangle	combined	square	rank	pentagon	combined	hexagon	rank	side
1	*12*	*j*	*13*	*d*	*14*	*h*	*15*	*w*	*2*
	. . .	*1*	. . . .	*2*	. . . . .	*3*	. . . . . .	*4*	
2	*221*	*w*	*535*	*t*	*741*	*y*	*951*	*yh*	*3*
	. . . . . .	*4*	. . . .	*8*	. . . .	*12*	. . . .	*16*	
3	*4321*	*y*	*7531*	*yw*	*10741*	*kb*	*12951*	*kh*	*4*
	. . . .	*9*	. . . .	*18*	. . . .	*27*	. . . .	*36*	
4	*54321*	*yh*	*97531*	*kh*	*1310741*	*lh*	*1713951*	*nh*	*5*
	. . . .	*16*	. . . .	*32*	. . . .	*48*	. . . .	*64*	
solid	**natural order**	**triangle**	**(difference) of two**	**triangle**	**(difference) of three**	**triangle**	**(difference) of its side in general**	**triangle**	**heteromeric**

Table 2: Amicable numbers.[29]

even-times-even	8	16	32
first element	7	[1]5	31
second element	5	11	23
third element	11	23	47
fourth element	71	287[30]	1,151
abundant element	220	2,024	17,296[31]
the larger deficient (one)	284	2,296	18,416

They were learnt from Nikomachos' *Introduction to Arithmetic*, which was translated into Arabic twice in the ninth century.

The first of these three themes is expounded in a long marginal text starting in the upper right-hand corner of the upper margin and running continuously to the end of the left half of the lower margin. The second theme is presented in tabular form for three cases explained by a preceding text. The third theme only appears in a table (Fig. 1 and Table 1). It has the form of a 10 × 10 square. It follows immediately after the text on the even and odd numbers and fills the right half of the lower margin. The table displaying Thābit b. Qurra's (d. 288 H/ 901 CE) rule for producing pairs of amicable numbers in three numerical examples (Table 2) and its faulty textual explanation appears in the Sackler manuscript 14 pages later, separated from the other number-theory material by the text on geometrical figures and their tabular documentation. Presenting information on number theory in one tightly knit package was clearly not Ẓāhir al-Dīn's intention. This speaks against considering teaching knowledge on this discipline as one of the purposes of the selected themes.

While the mistake in line 2 is an early scribal one, the numbers in the second row leave no doubt that whoever filled it in did not know what a prime number was and hence did not understand Thābit's rule, which says the following:

If $p_1 = 2^{n+1} - 1 + 2^n$, $p_2 = 2^{n+1} - 1 - 2^{n-1}$, $p_3 = 2^{n+1}(2^{n+1} + 2^{n-2} - 1)$ are three prime numbers greater than 2, then $a_1 = 2^n p_1 p_2$ and $a_2 = 2^n p_3$ are two amicable numbers.[32]

Both mistakes appear in all four manuscripts.[33] This suggests that Ẓāhir al-Dīn overlooked them when proofreading his collection. The first and foremost point of interest in the explanation is the wondrous character of amicable numbers, which bring the seeker and the sought-after together. That is why Plato called amicable numbers 'the magnet of the hearts' (*maghnāṭīs al-qulūb*), Ẓāhir al-Dīn believed.[34]

The subsequent lines instruct the reader to start from the class of numbers called 'even-times-even', that is, from powers of 2, and to calculate four basic elements. The first element is produced by subtracting 1 from the chosen power of 2, and the second by subtracting a quarter of that power of 2 from the first element, which then yields a prime number that is uneven. The third element is made by adding 'double'[35] (*sic*) the power of 2 to the first uneven number, the result of which is also an uneven number. The fourth element starts with the even number, to which an eighth of it is added. The result is multiplied by the even number itself and one is subtracted from the outcome. The two amicable numbers can be produced with these four elements. First, the second and third elements are multiplied. When multiplied by half of the even-times-even number, an 'abundant number' is the

[29] MS Cambridge, Mass., Harvard Arts Museums/Arthur M. Sackler Museum, 1984.463, fol. 58b.

[30] This is not a prime number as demanded by Thābit's rule. Hence the two resulting numbers are not amicable.

[31] This pair of amicable numbers was found in all likelihood by Thābit b. Qurra when he was working out his rule. See Hogendijk 1985 and Brentjes and Hogendijk 1989.

[32] Brentjes and Hogendijk 1989, 373.

[33] MS Tehran, Malik Library, 868, p. 87; MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 64b; MS London, British Library, Or. 12974, fol. 64a.

[34] MS London, British Library, Or. 12974, fol. 64a, upper margin, line 4.

[35] This is a mistake; it should actually be half the power of 2.

result, which is the smaller of the two amicable numbers. When the fourth element is multiplied by half of the even-times-even number (a deficient number), the bigger one is the result.[36] As mentioned before, this is only correct if the second, third and fourth elements are each a prime number greater than 2. However, the text does not make this clear, hence the mistaken result in the middle row.

2.2 Geometry

In the Safavid empire, geometry was taught at madrasas or in the homes of wealthy families. It primarily included three disciplinary fields: Euclidean geometry (*handasa*) following the *Elements* or shorter expositions such as Shams al-Dīn al-Samarkandī's *Ashkāl al-taʾsīs* and Qāḍīzāda al-Rūmī's commentary on Samarkandī's extract from Books I and II of the *Elements*; surveying (*misāḥa*) according to any of the numerous non-canonised texts on this branch of geometry; and spherical geometry on the basis of the *Middle Books*, as they were known (*Mutawassiṭāt*). Sometimes, higher-level texts such as Ṭūsī's edition of Apollonius's *Conics* were also taught.

In Ẓāhir al-Dīn's collection, two works are concerned with surveying, one combines algebra with surveying, two relate to subject matters in the *Elements*, a sixth one attempts to prove that the value of π is 3 1/7, a seventh one determines the chord if the sagitta is known, an eighth one relates to Nikomachos' *Introduction to Arithmetic*, a ninth one treats problems with inclined objects and a tenth calculates the volume of a fish pond. One appears as a marginal text running over several pages, five are brief marginal texts and two are interlinear texts in the centre of their respective folios, that is, between the main text of the riddles. The content of the long marginal text is partly summarised at the end of it in a table of plane and solid figures (Fig. 2). The texts talk about square roots and their squares as used in surveying and algebra, about geometrical definitions and about inclined objects. All this material provides basic information on elementary matters. I will only present two examples here to show that this knowledge reflects primary-school teaching, as in the case of number theory, algebra and systems of calculation.

The text that speaks about surveying and algebra serves to explain the terminology of both subdisciplines and their application to the same object, for instance 'side' for the unknown value x and 'quadrangle' for the square of the

[36] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 64b.

Fig. 2: MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 56a.

unknown value x^2 in surveying, and 'thing' and 'wealth' in algebra, providing a few numerical examples afterwards.[37]

The geometrical definitions and their subsequent table of drawn figures are taken from Ẓāhir al-Dīn's composition *al-Jāmiʿa*, which has been dealt with by Pourjavady and Rahimi-Riseh.[38] Although most of them resemble the definitions from the *Elements*, they are not a simple translation, but rather a free narration of their content. They begin with a statement that the indivisible (thing) – one of the attributes of the magnitudes – is the point, while the line is divisible in one direction, the plane in two and the mathematical body in

[37] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 37a, left margin, text in a rectangular frame.

[38] See the contribution of Pourjavady and Rahimi-Riseh in this volume.

three. A segment begins from two (points), followed by three and four. There are three kinds of lines – straight ones, that is, the extension provided by one's eyesight, then circular ones and curved ones. The straight ones can be in a ratio to each other, equal, parallel, cut each other, be tangential and so forth. There are ten well-known names for them: side; leg; foot-point of a height; height; basis; side (direction); diagonal; chord; sagitta/axis; and elevation. As for planes, Ẓāhir al-Dīn enumerates convex and concave planes as well as even ones. The mathematical solids he describes are either formed by plane areas and lines or by plane areas enveloping them. Before he enumerates selected planes and solid figures, he introduces the Euclidean definitions that the plane area is the end of the solid, the line of the plane area, the point of the line and the limit of the geometrical figure. From the plane figures, he first elaborates on the circle with its centre, the diagonal, the chord, the basis of a segment, the arc and the sagitta. Then he turns to trigonometric magnitudes like the sinus and the Sinus Versus (Cosinus). Afterwards he returns to naming curved plane figures like the segment, the crescent, the egg and composite ones before moving on to plane figures formed by straight lines and their angles, like triangles, quadrangles, polygons or figures with unequal sides.

The final part of the text is dedicated to a variety of solid figures, in particular the cylinder, the pyramid and the truncated pyramid, cones, the sphere and related magnitudes from spherical geometry, as well as polyhedrons.[39] The table follows after the text, with images of plane and solid figures.[40] To document the social acceptability of geometry, Ẓāhir al-Dīn ascribes the creation of a spherical universe ranging from the highest sphere to the lowest element (Earth) in relation to God. The fact that Tabrīzī, the calligrapher, wrote this ascription in a style resembling a verse from the Qurʾān underlines the attribution's social function.[41]

While the text is simple, its language highlights the fact that Ẓāhir al-Dīn did not merely recapitulate items he had learnt by heart. The result is not particularly systematic, but its peculiarities show the author's independence from standard school texts, which followed a different order of presentation; they do not include all the figures and subjects mentioned by Ẓāhir al-Dīn and they use different technical terms at times.

2.3 Algebra and arithmetical operations

Algebra is dealt with in two interlinear texts between the lines of the riddles in the centre of their respective pages. The first of them is dedicated to the names of unknown quantities in the fields of surveying and algebra, as mentioned above. The second text outlines different methods for determining unknowns. These methods include the well-known *regula falsi*, algebra, analysis, proportions and basic arithmetical operations.[42] The surprising feature of this second text compared to the pieces on number theory and geometry summarised so far is that it is rather lengthy, explaining the rules of each method step by step and providing examples as well. This type of text actually has no place in a riddle-solving exercise because it suggests that the participants at the banquet would first have to learn the methods before they could even think about whether they had any relation to the riddle in the middle. If this had been the case, the banquet would have been over before the participants had solved any of the riddles. But if they were already familiar with the methods, they would not have needed such basic explanations and exercises. This is another confusing aspect of an already riddle-laden collection. However, a comparison with the other manuscripts accessible to me revealed that the text is not in all of them in its entirety.[43] This is particularly so for MS Tehran, Malik Library, 868 where the text ends with '...' and has no exercises at all. If this difference does not reflect an incomplete exemplar of the Malik 868 manuscript, it may be that a later client who was less familiar with the mathematical methods mentioned above had wanted them to be explained and accompanied by exercises. Should such an extension of what was originally a much shorter summary of methods be endorsed by further research, it would confirm that individuals who commissioned a later copy actively interfered with the collection's composition, as Kia has shown in his contribution to this volume.[44]

The examples add a further surprising feature as they are taken from what is known as 'recreational mathematics'. In other words, they are little riddles in themselves. They invite the newcomer to use mathematical methods to figure out (or

39 MS Istanbul, Süleymaniye Library, Ayasofya 4785, fols 59b–60b.

40 MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 60b.

41 MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 60a; MS Cambridge, Mass., Harvard Arts Museums/Arthur M. Sackler Museum, 1984.463, fol. 55b.

42 MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fols 64b–70a; MS Istanbul, Süleymaniye Library, Ayasofya 4785, fols 71a–75a.

43 The corresponding text in MS Tehran, Malik Library, 868, p. 103 is much shorter. MS London, British Library, Or. 12974, fols 70b–73b lacks the end of the text and does not contain all of the examples because two folios are missing.

44 See the contribution of Kia in this volume.

guess) a number after several operations have been carried out on it. Here is an example:[45]

> Question: What is the number? If one adds its fourth to it and its fifth to the result and then subtracts the cube of the prime 2 from the compound, then the cube 8 of the given prime is left.

Some examples leave the domain of arithmetic and algebra and move into unrelated parts of the astral sciences. In the following case, the explanation of the enigmatic connection between the two fields and the astral sciences is the implicit demand to know that the number of the Ptolemaic constellations is 48:[46]

> Which plane number equals the images of the imagines among the observed stars after its third and its fourth have been multiplied?
> [Answer:][47] The product of a third and a fourth is half of a sixth. 48 is known. The result is the unknown 'wealth' (that is x^2) 576. Its root is 24, [which is] the number sought.
> This is because one says: a half and a third and its fourth equals twenty-six.[48] The known one is 26. The common denominator is 12[49]. The known 26 was multiplied by the common denominator 12. The result is divided by the given fractions, 13.[50] The outcome is 24, the [root of the] unknown 'wealth'.

The last line of the interlinear text on fol. 69b of the Sackler manuscript, which is not the last line of the texts written between the lines of the riddles, contains a catchword, *ḍilʿ* ('side'). It usually indicates the continuation of the text on the following page,[51] but this is not the case here. Rather, it refers to the marginal text on the same page, starting in the left corner of the upper margin. The remainder of the texts

Fig. 3: MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 66a, left margin.

on methods appears in the right corner of the upper margin of fol. 70a. This textual positioning makes it even harder to understand the hidden rules of the game for any participant at the banquet or any reader like myself.

An analogous shift from the interlinear to the marginal text is found in the Ayasofya manuscript. In the Malik 868 manuscript, the short version of this text is only in the margin.[52] In the British Library manuscript, the incomplete text appears between the lines of the riddles.[53] This indicates that the placement of at least some of the explanatory texts had no particular significance.

Arithmetical operations are discussed in the text itself and are visualised in tables and diagrams. The text also teaches the nine common fractions, which Ẓāhir al-Dīn says are the invention of ʿAlī, the son-in-law of Prophet Muḥammad and the first Shīʿī Imām.[54]

A related diagram shows the attribution of numbers to the parts of the fingers of the right hand (Fig. 3).[55] Counting starts with one on the lowest part of the little finger. It runs consecutively along the parts of each finger, i.e., up to three on the little finger, from four to six on the ring finger and so

[45] MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 65b, interlinear text, lines 9–11.

[46] MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 67b, interlinear text, lines 3–4.

[47] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 74a, interlinear text, lines 8–74b, marginal text, upper margin, line 6 and MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 67b, interlinear text, lines 4–10.

[48] These fractions refer to 24.

[49] MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 67b wrongly says 16 here. MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 74b, upper margin, line 5.

[50] This means the sum of the numerators of the three fractions.

[51] See the contribution of Pourjavady and Rahimi-Riseh in this volume.

[52] MS Tehran, Malik Library, 868, p. 103.

[53] MS London, British Library, Or. 12974, fols 72b–75b.

[54] MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 70a, interlinear text; MS London, British Library, Or. 12974, fol. 69b, interlinear text.

[55] MS Tehran, Malik Library, 868, p. 103, left margin; MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 70b, right margin; MS London, British Library, Or. 12974, fol. 69b, right margin.

forth. Once 10 has been reached on the lowest part of the index finger, the next two parts on that finger are 20 and 30, followed by 40 and 50 on the thumb. 60 to 90 are placed on the tips of the four fingers and 100 on the palm.

All in all, an exploration of the sample texts and tables of the three selected mathematical fields confirms that the purpose of their inclusion in the collection cannot have been educational. The knowledge addressed is very elementary. But even so, the copyist did not regard it as his task to ensure that it was correct despite the fact that at least one of the manuscripts he produced was destined for the shah's library. Apparently, the riddles were solvable even if the shorter or longer pieces of knowledge were imprecise. They therefore differ significantly from today's crossword puzzles and similar games.

Moreover, it seems that the enigmatic references to mathematical terms or concepts did not necessarily have to be matched by a piece of mathematical knowledge written in the margin or in the lines between the wording of the riddle, be it a text, a table or a diagram. The references to the odd and 'even-times-even' numbers on fol. 55b of the Ayasofya manuscript and the cube – the first of the solids to be found (in a riddle?) on fol. 56a of the same copy – are such examples. Neither of them has any correspondence in the other parts of those two pages. Some expressions in the riddle on fol. 55b are linked to a more philosophical text in the margins, which discusses the ranks of the numbers, among other things. A table on fol. 56a lists figurative numbers, but for planar figures, not solid ones. A brief text on eight being the first solid number, appropriated from the first chapter of the *Rasāʾil* (*Epistles*) of the *Ikhwān al-Ṣafāʾ*, appears in the right-hand margin on fol. 55b. It would have corresponded to the reference on fol. 56a. An even more confusing case can be seen in the discussion of the seven seas in the margins of fol. 54b because there is not a single word of reference to the seas in the riddle on this page. Nor does a reference to them appear immediately before or after this page.

The relative independence of the riddles and the texts, tables and diagrams elsewhere on each page is also reflected in the fluidity of their position. Diagrams and tables, for instance, appear in different locations in the four analysed manuscripts. The same diagram can be found in a margin in one manuscript, while in another it appears between the lines of a riddle. Diagrams and tables were put on different sides of a folio and in different corners or other parts of the margins. This is the case for the diagrams of a planetary orb, lunar

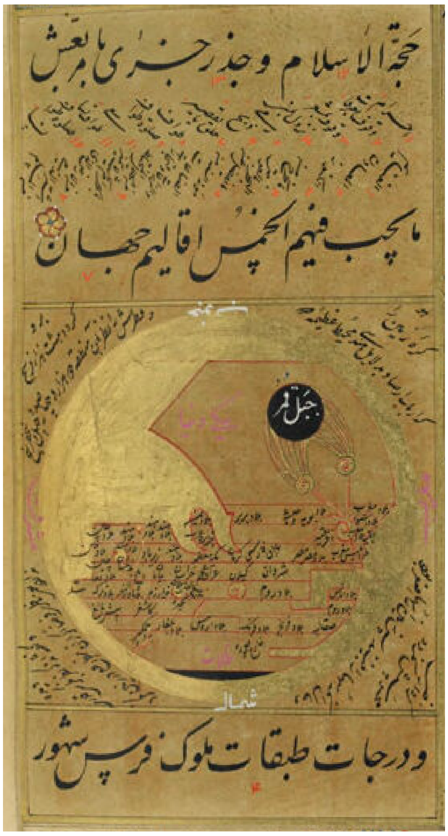

Fig. 4: MS Cambridge, Mass., Harvard Art Museums/Sackler Art Museum, 1984.463, fol. 104b, detail.

phases, the numerical division of the right hand, the process of seeing or the tables of the correspondences between letters and numbers and those of figurative or amicable numbers and of geometrical figures, for example.[56]

2.4 Geography

The contributions to geography are of a similar kind. They include a fair number of mistakes, but in some parts, they also reflect short bits of new(er) knowledge. They are presented in an unusual tabular form in one case.

The geographical texts, tables and map appear in the last part of the collection. The map depicts the Old World and has been put in the centre between the lines of the riddles (Fig. 4).[57]

[56] MS Tehran, Malik Library, 868, pp. 2, 44, 63, 87, 100, 119 (the table with the figurative numbers and the geometrical table are missing); MS Istanbul, Süleymaniye Library, Ayasofya 4785, fols 25b, 39b, 51b, 56a, 60b, 64b, 70b, 78a; MS London, British Library, Or. 12974, fols 18b, 39a, 52a, 56a, 60a, 64a, 69b (the diagram of the lunar phases seems to be missing); MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fols 11b, 33a, 46a, 51a, 56a, 58a, 66a, 76a.

[57] MS Tehran, Malik Library, 868, p. 175; MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 103a; MS London, British Library, Or. 12974, fol. 105b.

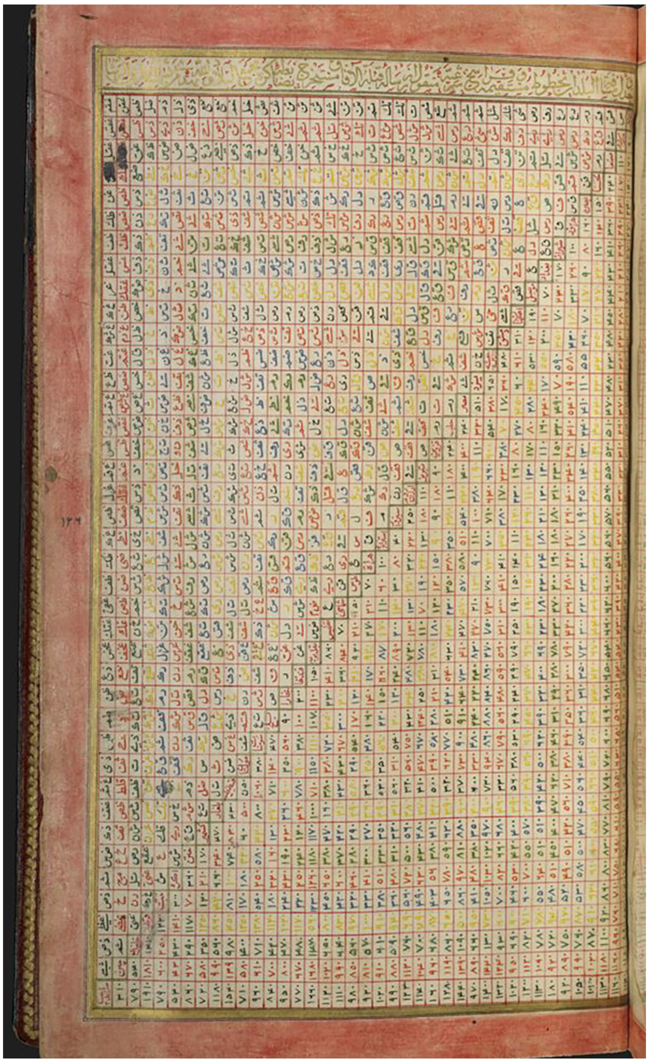

Fig. 5: MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 126a.

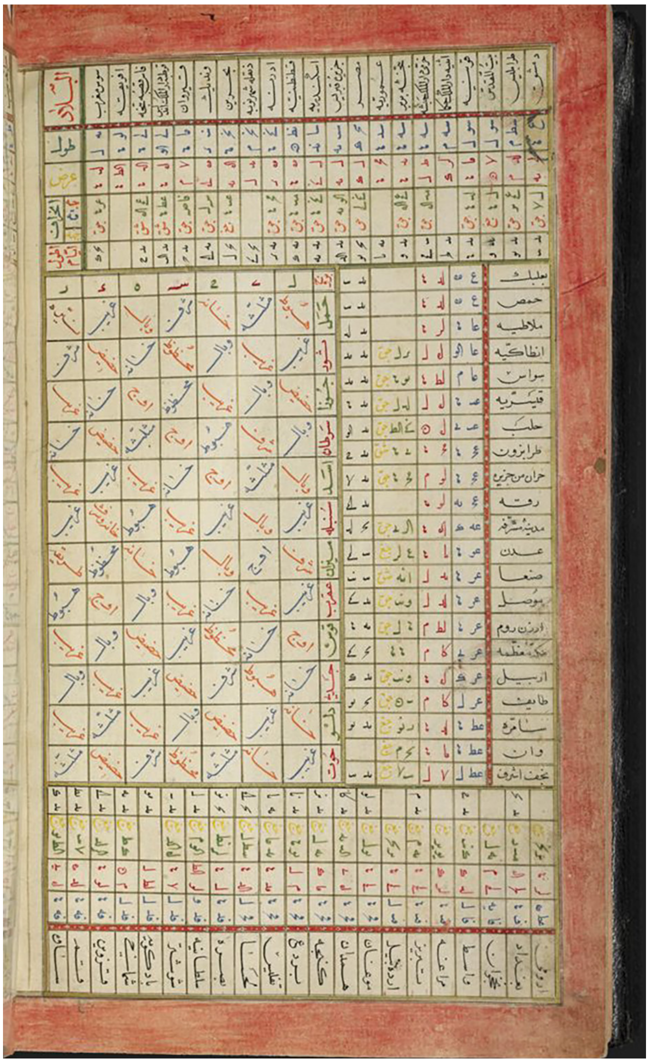

Fig. 6: MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, table on fol. 126b.

In the four corners around it, a text elaborates on the sphericity of the Earth and the seven climes.[58]

A few pages later, a diagram of the layout of the Kaʿba in Mecca is briefly elucidated by a few remarks on the length of its sides.[59] There is a paper instrument on the same page called the Indian circle that explains how the reader can find the direction in which Mecca lies in order to pray.

Further geographical tables appear about ten pages later. A short table informs the reader about the seven climes.[60] It is followed by a table containing distances in Persian miles (*farsakh*) along straight lines between 40 cities or larger regions taken from an anonymous *Risālat qiblat al-āfāq* (Fig. 5).[61] It covers an entire page, thus deviating quite clearly from the main layout forms used in all the manuscripts. The last geographical item is a long table stretching along the margins over two pages (Fig. 6).[62] It provides the latitude and

[58] This only applies to MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 104b.

[59] MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 108b.

[60] MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 125b.

[61] This description reflects the table's title. MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fol. 126a; MS Istanbul, Süleymaniye Library, Ayasofya 4785, fol. 121a; MS London, British Library, Or. 12974, fol. 124a.

[62] MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463, fols 126b–127a. In MS London, British Library, Or. 12974, this table appears 12 folios before the table presenting distances for countries, cities and towns, i.e., fols 112b–113a and 124a. In MS Istanbul, Süleymaniye Library, Ayasofya 4785, fols 110b–111a, this table also precedes the distance table by 12 folios. In addition, the table on the seven climes appears in different positions in the three manuscripts in relation to the two tables discussed here: in Ayasofya 4785 and British Library, Or. 12974, it immediately precedes the distance table, while in the Sackler manuscript it precedes the latitude-longitude-qibla table. MS Tehran, Malik Library 868 has none of these tables in it.

longitude of 126 cities together with the angle identifying the respective direction of prayer for each of them.

The notable characteristics of the map (Fig. 4) and the two geographical tables shown in Figures 5 and 6 are their connection to genres of the astral sciences on the one hand and their inclusion of a few new or even unknown elements on the other. The map, for instance, depicts a much larger African continent than the image found in most other pre-modern Arabic, Persian and Ottoman Turkish world maps. It also emphasises Cape Horn prominently. The only other map of this type known so far is from a copy of Niẓām al-Dīn al-Nīsābūrī's (d. 729 H/1328–29 CE) commentary on Ṭūsī's *Tadhkira fī ʿilm al-hayʾa* ('Memoir on the Science of the Configuration [of the Universe]').[63] What is more, all the copies of this map in the four manuscripts in Ẓāhir al-Dīn's collection bear the Turkish inscription *yeni dünyā* ('New World'), but it is wrongly positioned – in Africa, close to Cape Horn – and they fail to show any territories beyond the classical world. Since no further documentary evidence of knowledge about the New World in Safavid manuscripts is known from Safavid courtly circles, I am inclined to assume that Ẓāhir al-Dīn acquired this piece of knowledge orally, namely from Ottoman or European travellers.

The table of the longitudes, latitudes and prayer directions to Mecca appears to be copied from post-ninth century H/fifteenth-century CE material. Not surprisingly, a systematic comparison of the table's data with the information in such works produced in Iran and Central Asia up to the middle of the fifteenth century CE yields a mixed character for Ẓāhir al-Dīn's possible source. Indeed, 39 of the 126 entries coincide with the values in an anonymous Timurid table of longitudes, latitudes and qibla directions from the early tenth century H/beginning in 1494 CE.[64] They confirm David King's observation that the majority of such tables known to him from the sixteenth to the nineteenth century depended directly or indirectly on this source.[65]

Another 30 cases are standard scribal mistakes and should be added to the items of data from the Timurid table. A fair number of the *qibla* values are significantly different, even if the longitude and latitude agree fully or almost fully with the figures mentioned in the Timurid table. It is unclear whether at least some of these substantial differences are merely scribal mistakes or whether they reflect some merger with another source that was possibly misread or in disorder in terms of its material.

The cases with substantial differences amount to 45. One member of this group goes back to the new astronomical handbook (*Zīj*) produced in Samarqand under the auspices of the Timurid prince Ulugh Beg (r. 812–853 H/1409–1449 CE) and two others go back to the slightly earlier work compiled by Ghiyāth al-Dīn Kāshī (d. 832 H/1429 CE), one of the collaborators of Ulugh Beg.[66] In other cases, however, the values either deviate from Kāshī's new results or are not found in his tables and those of Ulugh Beg. This indicates that whoever assembled Zāhir al-Dīn's table did not directly rely on these two Timurid sources. Somewhat surprisingly, the *qibla* values for Yazd and Sabzavar provided in this table are of a better quality than the version of the Timurid table published by King.[67] This indicates that the chain of tables that led to Zāhir al-Dīn's variant started from a rather 'more correct' version of the Timurid table of longitudes, latitudes and *qibla* directions than the extant copy. All in all, these observations demonstrate that neither Zāhir al-Dīn nor any of the people who worked on this information verified and then corrected the table's data. Since the table is not placed around a central riddle text, this lack of accuracy may not have disturbed any of the banquet's guests when the collection was accessed and the participants tried to solve some of its riddles.

The geographical table about distances between regions and cities presents the locations along its diagonal, starting from the island of Sarandīb (Śri Lanka?) and ending with Medina und Mecca. In the upper triangular half, it presents *abjad* numbers, that is, letters according to their numerical values. In the lower half, the numbers are presented in the eastern form of Indo-Arabic numerals. The order of the regions and cities is unsystematic: at the beginning, it goes from the Indian Ocean to China and from its capital Khānbalıq to Córdoba, jumping from there to Tibet, Agra (India), Khotan (now North-west China) and Kashmir (Pakistan/India). Then follow the Bulghars (in the Volga basin), Constantinople (Istanbul), Multan (Pakistan), Somnath (India), Balkh (Afghanistan), Bukhara (Uzbekistan), Qandahar (Afghanis-tan) and Baqala Bariyya (the Baqala Desert, in Abu Dhabi?), moving on to Tiflis in the Caucasus Mountains. In a similar manner, the

[63] I owe this information to F. J. Ragep.

[64] King 1999, 456–471.

[65] I would like to thank David A. King for this information.

[66] King 1999, 469–470, entries 187, 192 and 199.

[67] King 1999, 464, entry 122 and 467, entry 159.

Fig. 7: MS Tehran, Malik Library, 868, p. 100.

Fig. 8: MS Tehran, Malik Library, 868, p. 72.

further part of the table oscillates between cities in Iran, Iraq, Syria, Egypt, Saudi Arabia and Yemen. The numerical data suggests that the point of orientation was Mecca. But even so, the distances in the lowest line do not increase continuously from Mecca in the lower right-hand corner to 1140 in the lower left-hand corner or rise in regular steps from Sarandīb in the upper left-hand corner to 1140 in the lower left.

3. Forms of entering and representing mathematical knowledge

The almost aphoristic nature of numerous mathematical writings in Ẓāhir al-Dīn's collection suggests that the primary reason for their inclusion was not to provide the guests at a banquet with some new mathematical knowledge as part of the evening's entertainment. In general, it seems their primary purpose was to fill the empty space around the long riddle and between its lines in a way linked to words used in the enigmatic text.

Relatively long or big items such as the map, the extensive geographical tables and the survey of geometrical figures speak for their inclusion in the collection on account of their size and complexity, but not just because of spatial necessity. As suggested earlier, the extensive geographical tables may be additions desired by a client who commissioned a later copy, probably after the two Malik manuscripts were produced. The layout of their pages clearly differs from the bulk of the pages in each copy. They do not contain any texts at all in the centre or in the margins. Thus, they were not meant to contribute to riddle-solving, but served to round off the overview of respectable mathematical knowledge. The map and the survey of the geometrical figures, on the other hand, were most likely part of the original compilation since they are found in all five manuscripts. They contain basic knowledge that Ẓāhir al-Dīn obviously wished to have included as representations of the two specific fields

Fig. 9: MS Tehran, Malik Library, 868, p. 29.

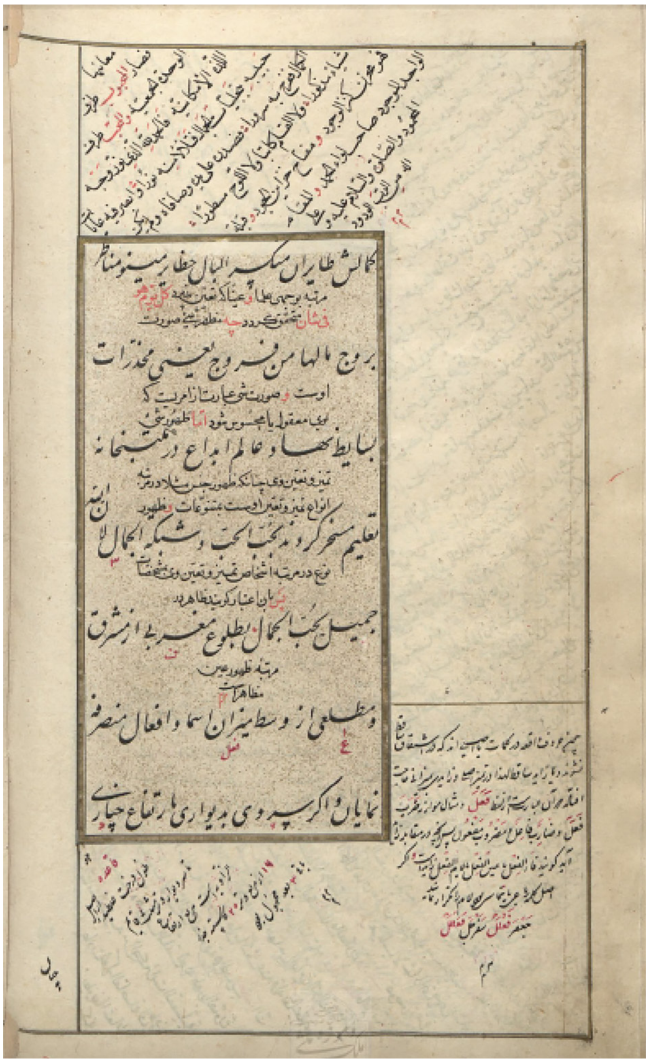

Fig. 10: MS Tehran, Malik Library, 1571, fol. 77b.

of knowledge. Structurally, they tally with each other in so far as they provide short bits of information listed in one or more sequences. This sets them apart from texts that provide explanations and examples, like the case discussed in the previous section.

Other shorter texts or smaller tables and diagrams may have been included in the collection because of the compositional and layout challenges, however. This speculation can be tested by analysing the arrangements of the copies in the Malik 868 manuscript and its older parallel, MS Tehran, Malik Library, 1517. Both copies are incomplete, resemble each other strongly and thus provide insights into the calligrapher's approach when he was reproducing the individual pages. My attempt to determine which rules the calligrapher Muḥammad Shafīʿ Tabrīzī (d. after 1097 H/1686–87 CE) followed in this process brought very few conclusive results to light, though.

The most clearly visible feature in both manuscripts is the privileged treatment of the riddle itself. Judging by the flow of the handwriting, it was written down continuously from beginning to end. No such regularity can be observed for the interlinear or marginal texts, however: sometimes they are present and sometimes they are not. The same applies to the mathematical and astral diagrams and tables in the manuscripts (Fig. 7).

On the other hand, the numerous numbers and letters symbolizing numbers as well as words or textual snippets were entered below or above the lines of the riddle, encroaching upon the space that was taken up by the interlinear texts later (Fig. 8).

The placement of all these components of the collection even varies between these two early copies. The calligrapher obviously could not see any clear or binding pattern for entering and positioning the four elements of the collection.

This flexibility in relating items in the four categories to the riddle is also apparent in the three later manuscripts, although they were clearly executed much more carefully and thoroughly.

Another rather surprising feature of Tabrīzī's work is his lack of use of the available space (Fig. 9). In the two early copies, he obviously still had difficulties estimating how much space any given piece might cover. This is particularly easy to see in the texts he put in the margins and in the central riddle. The texts in the margins were often finished with some blank space left, but not enough to be able to add a new snippet (Fig. 10). In the other three copies, some of these empty spaces were filled with floral or other decorative patterns. There were no such additions in the two early copies, which are incomplete, but lines often indicate the end of an allotted space. Separators like these were also added in other cases. However, sometimes they left more space than the scribe actually required for a diagram, text or table.

In the centre, the lines of the riddle often contain more words than a line otherwise has (Fig. 8). Writing a few letters, half a word or occasionally even a long word above the last word in a line was normal scribal practice. But in the case of the two early manuscript copies, this practice goes beyond the usual limits as two, three or even four words are placed one above the other at the end of a line. Occasionally, there was not enough space for them even then, so the scribe wrote over the frame or added a longer phrase in the small margin outside it. In both these cases of non-standard scribal behaviour, further research and reflection is needed to make sense of the scribe's decisions to pick unconventional or inaesthetic solutions. He could easily have chosen an alternative such as dividing the available space into portions first with a ruler, continuing the overflowing words of the riddle on the next line or using a smaller size for the script in which he copied the riddles.

The unusual work procedures reflected in the two incomplete copies of Ẓāhir al-Dīn's collection do not only apply to scribal activities; they are also visible in the decorative elements. A considerable amount of decorative work was performed before the calligrapher had finished his duties. The many minor decorative pieces found in the later manuscript copies are largely absent from the earlier ones except for the small golden floral spots spread across the space in the central rectangle around the riddle.

4. Conclusions

The collection of manuscripts that Ẓāhir al-Dīn allegedly commissioned as the result of a fancy banquet with other members of the Safavid elite is a fascinating, highly challenging cultural product. It brings together entertaining word, number and letter games, which were not primarily intended to evoke precise answers, but to get the participants to use methods associated with a broad range of fields of knowledge, cultural practices and bodily movements. The mathematical parts chosen for investigation for this paper turned out to be truly elementary, replete with mistakes and not requiring any advanced knowledge of the subject. If the one text that not only summarises such standard knowledge, but teaches methods was, indeed, a later addition to the collection, then it seems that one of the interested elite clients apparently wished to go beyond the mere repetition of examples and rules familiar from teaching at home or school. Another client seems to have favoured tabulated geographical and astrological information and asked for it to be added to the collection without it being explicitly connected to the riddles. Mathematical and geographical terms are abundantly present in the riddles as well. Their exact relation to the accompanying texts, tables and diagrams in the manuscripts calls for further scrutiny in future.

MANUSCRIPTS

MS Cambridge, Mass., Harvard Art Museums/Arthur M. Sackler Museum, 1984.463.

MS Istanbul, Süleymaniye Library, Ayasofya 4785.

MS London, British Library, Or. 12974.

MS Tehran, Malik Library, 868.

MS Tehran, Malik Library, 1571.

REFERENCES

Primary Sources

Āmulī, Shams al-Dīn, *Nafāyis al-funūn fī ʿarāyis al-ʿuyūn*, ed. Abū l-Ḥasan Shaʿrānī and Ibrāhīm Miyanjī, vol. 3, Tehran: Čāp-i islāmiyya,1379 HS/2001.

Ibn Sīnā, Abū ʿAlī al-Ḥasan, *Tisʿ rasāʾil fi l-ḥikma wa-l-ṭabīʿiyyāt*, Qusṭanṭīniyya: Maṭbaʿat al-Ǧawāʾib, 1298 H/1881.

Ṭūsī, Naṣīr al-Dīn, *Akhlāq-i nāṣirī*, ed. Mujtabā Mīnuvī and Aḥmadriḍā Ḥaydarī, Tehran: Khwārazmī,1356 HS/1977.

Secondary Literature

Brentjes, Sonja and Jan P. Hogendijk (1989), 'Notes on Thabit ibn Qurra and his Rule for Amicable Numbers', *Historia Mathematica*, 16/4: 373–378.

Hogendijk, Jan P. (1985), 'Thabit ibn Qurra and the Pair of Amicable Numbers 17296, 18416', *Historia Mathematica*, 12/3: 269–273.

Kia, Chad (2024), 'Adjunct Texts: Five Glosses on a Page in a Safavid Collection', *manuscript cultures*, 22: 127–136.

King, David A. (1999), *World-maps for Finding the Direction and Distance to Mecca: Innovation and Tradition in Islamic Science*, Leiden: Brill.

Melvin-Koushki, Matthew (2017), 'Powers of One: The Mathematicalization of the Occult Sciences in the High Persianate Tradition', *Intellectual History of the Islamicate World*, 5/1: 127–199.

——, Matthew (2021), 'The Occult Sciences in Safavid Iran and Safavid Occult Scientists Abroad', in Rudi Mathee (ed.), *The Safavid World*, London: Routledge, 403–427.

Mousavi, Razieh and Hamid Bohloul (2024), 'Manifestation in the One: Singularity of Knowledge in Mīrzā Ibrāhīm's *Lughaz-i marqūm*', *manuscript cultures*, 22: 155–168.

Pourjavady, Reza and Ahmad-Reza Rahimi-Riseh (2024), 'The Late Safavid Riddle Codices Portrayals of Ẓahīr al-Dīn Mīrzā Ibrāhīm's Worldview', *manuscript cultures*, 22: 5–26.

Stephenson, John (1923), 'The Classification of the Sciences according to Nasiruddin Tusi', *Isis*, 5/2: 3329–3338.

Vesel, Živa (1986), *Les encyclopédies persanes: essay de typologie et de classification des sciences* (Mémoire, 57), Paris: Éd. recherche sur les civilisations.

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